Essentials of Calculus

Homework 4.7

Logistic growth

1. The number of fish in a pond in t years is given by

$$y = \frac{1000}{1 + 9e^{-0.5t}}$$

fish.

- Sketch a graph of the fish population as a function of years.
- What is the carrying capacity of the pond?

  Numeric answer: The carrying capacity is 1000 fish.
- What is the current fish population of the pond?

  Numeric answer: There are currently 100 fish in the pond.
- How many fish will be in the pond in 5 years?

  Numeric answer: There will be 575 fish.

2. The number of bacteria in a dish in t days is given by

$$y = \frac{100000}{1 + 99e^{-2t}}$$

fish.

- Sketch a graph of the bacteria population as a function of days.
- What is the carrying capacity of the dish?

  Numeric answer: The carrying capacity is 100000 bacteria.
- What is the current bacteria population of the dish?

  Numeric answer: There are currently 1000 bacteria in the dish.
- How many bacteria will be in the dish in one day?

  Numeric answer: There will be 6945 bacteria.

3. An island contains 100 rodents, which have a growth rate of 0.2/year. The island has a carrying capacity of $M = 10000$ rodents.

- Find a formula $P(t)$ for the rodent population in t years.

  Numeric answer: $P(t) = \frac{1000000}{100+9900e^{-0.2t}}$

- How many rodents will be on the island in 3 years?

  Numeric answer: There will be 181 rodents.

4. A lake contains 200 frogs, which have a growth rate of 0.1/year.
   - Assuming exponential growth, how many frogs will there be in 10 years?

     Numeric answer: There will be 544 frogs.
   - Assume that the lake has a carrying capacity of 5000 frogs. write down a formula for the population $P(t)$ of frogs in t years.

     Numeric answer: $P(t) = \frac{5000}{1+24e^{-0.1t}}$
   - Assuming logistic growth, how many frogs will there be in 10 years?

     Numeric answer: There will be 509 frogs.
5. A jar contains 500 bacteria, which have a growth rate of 2/month.
   - Assuming exponential growth, find a formula $P(t)$ for the number of bacteria in t months.

     Numeric answer: $P(t) = 500e^{2t}$
   - Assuming exponential growth, how many bacteria will be in the jar in 12 months?

     Numeric answer: There will be $1.32 \cdot 10^{13}$ bacteria.
   - Assume that the jar has a carrying capacity of 10000 bacteria. write down a formula for the population $P(t)$ of bacteria in t months.

     Numeric answer: $P(t) = \frac{10000}{1+19e^{-2t}}$
   - Assuming logistic growth, how many bacteria will be in the jar in 12 months?

     Numeric answer: There will be 10000 bacteria.
6. A mountain range contains 50 yeti, which have a growth rate of 0.05/year.
   - Assuming exponential growth, write down a formula $P(t)$ for the yeti population in t years.

     Numeric answer: $P(t) = 50e^{0.05t}$
   - Assuming exponential growth, how many yeti will be in the range in 50 years?

     Numeric answer: There will be 609 yeti.

- Assume that the mountain range has a carrying capacity of 1000 yeti. write down a formula for the population $P(t)$ of yeti in t years.

  Numeric answer: $P(t) = \frac{1000}{1+19e^{-0.05t}}$

- Assuming logistic growth, how many yeti will there be in 50 years?

  Numeric answer: There will be 391 yeti.

7. Assume that the human population is 7 billion and that we have a growth rate of 0.01/year.
   - Assuming exponential growth, write down a formula $P(t)$ for the human population in t years.

     Numeric answer: $P(t) = 7e^{0.01t}$ billion.

   - Assuming exponential growth, how many people there be in 2025?

     Numeric answer: There will be 7.36 billion people.

   - Assume that the world has a carrying capacity of 100 billion people, write down a formula for the population $P(t)$ in t years.

     Numeric answer: $P(t) = \frac{100}{1+13.29e^{-0.01t}}$

   - Assuming logistic growth, how many people will there be in 2025?

     Numeric answer: There will be 7.33 billion people.