Essentials of Calculus

Homework 4.3

Global maxima and minima

1. Find the absolute maxima and minima of the following functions on the given interval.

   a) $f(x) = 2x^2 + 8x - 1$ for $-1 \leq x \leq 5$.

   Numeric answer: The minimum is at $x = -1$, with a value of $f(-1) = -7$
   The maximum is at $x = 5$, with a value of $f(5) = 89$

   b) $f(x) = x^3 - 12x^2 + 3$ for $0 \leq x \leq 4$.

   Numeric answer: The maximum is at $x = 0$, with a value of $f(0) = 3$
   The minimum is at $x = 4$, with a value of $f(4) = -125$

   c) $f(x) = \sqrt{x} - x$ for $0 \leq x \leq 9$.

   Numeric answer: The maximum is at $x = 1/4$, with a value of $f(1/4) = 1/4$
   The minimum is at $x = 9$, with a value of $f(9) = -6$

   d) $f(x) = x^4 - 4x^3 + 1$ for $-1 \leq x \leq 5$.

   Numeric answer: The maximum is at $x = 5$, with a value of $f(5) = 126$
   The minimum is at $x = 3$, with a value of $f(5) = -26$

2. For each part, sketch the graph of a function meeting the given conditions.

   a) $f'(x) = 0$ for $x = 1, 3$, $f''(x) = 0$ for $x = 2$.
   $f(1) = 3$, $f(2) = 2$, $f(3) = 0$
   f' and f'' have the following sign charts.

x		1	2	3	
$f'(x)$	+	0	−	0	+
$f''(x)$	−	−	0	+	+

b) $f'(x) = 0$ for $x = 2$, $f''(x) = 0$ for $x = 2$.
$f(2) = 1$
f' and f'' have the following sign charts.

x		1	2	3	
$f'(x)$	+	+	0	+	+
$f''(x)$	−	−	0	+	+

c) $f'(x) = 0$ for $x = 1, 3$, $f''(x) = 0$ for $x = 1, 2$.
$f(1) = 3$, $f(2) = 0$, $f(3) = -1$
f' and f'' have the following sign charts.

x		1	2		3	
$f'(x)$	−	0	−	−	0	+
$f''(x)$	+	0	−	0	+	+

3. Sketch the graphs of the following functions. (The domain is the entire real line.)

   a) $f(x) = 2x^2 - 6x + 2$

   b) $f(x) = x^3 - 6x^2$

   c) $f(x) = x^4 - 4x$

   d) $f(x) = x^3 - 3x^2 + 3x + 2$

4. Find the absolute maxima (or state that it doesn't exist) or minima (or state that it doesn't exist) for the following functions. (If no interval is given, assume the domain is the entire real line.)

   a) $f(x) = xe^x$

   Numeric answer: There is no maximum,
   The minimum is at $x = -1$, with a value of $f(-1) = -e^{-1}$

   b) $f(x) = \ln(x) - x$ for $x > 0$.

   Numeric answer: The maximum is at $x = 1$, with a value of $f(1) = -1$
   There is no minimum.

c) $f(x) = \frac{x^2}{1+x^2}$

Numeric answer: There is no maximum,
The minimum is at $x = 0$, with a value of $f(0) = 1$

d) $f(x) = \frac{x}{1+x^2}$

5. For each of the following profit functions, find the quantity q which will maximize profits.

a) $P(q) = 288\sqrt{q} - q + 50$ dollars.

Numeric answer: The profit is maximized at $q = 20736$

b) $P(q) = 100q - 2q^2 - 5$ dollars.

Numeric answer: The profit is maximized at $q = 25$