# Essentials of Calculus

## Homework 3.4

## The product and quotient rules

1. Find the derivative of each of the following functions.

   a) $f(x) = xe^x$

   **Numeric answer:** $f'(x) = xe^x + e^x$

   b) $f(x) = x^2(x^2+1)^5$

   **Numeric answer:** $f'(x) = 2x(x^2+1)^5 + 10x^3(x^2+1)^4$

   c) $f(x) = x^2e^{x^2}$

   **Numeric answer:** $f'(x) = 2xe^{x^2} + 2x^3e^{x^2}$

   d) $f(x) = \frac{x^2}{x^2+2x+1}$

   **Numeric answer:** $f'(x) = \frac{2x^2+2x}{(x^2+2x+1)^2}$

   e) $f(x) = \frac{e^x}{e^x+1}$

   **Numeric answer:** $f'(x) = \frac{e^x}{(e^x+1)^2}$

   f) $f(x) = (xe^x+1)^3$

   **Numeric answer:** $f'(x) = 3(xe^x+1)^2(xe^x+e^x)$

   g) $f(x) = x^2\ln(x)$

   **Numeric answer:** $f'(x) = 2x\ln(x) + x$

   h) $f(x) = \frac{\ln(x)}{x}$

   **Numeric answer:** $f'(x) = \frac{1-\ln(x)}{x^2}$

   i) $f(x) = \frac{2x^2-3x+1}{x^2+3}$

   **Numeric answer:** $f'(x) = \frac{3x^2+10x-9}{(x^2+3)^2}$

   j) $f(x) = e^x\ln(x)$

   **Numeric answer:** $f'(x) = e^x\ln(x) + \frac{1}{x}e^x$

2. Let $f(x) = \frac{4x-1}{x+1}$. Find the tangent line to the graph $y = f(x)$ at $x = 0$.

   **Numeric answer:** The tangent line is $y = 5x - 1$

3. Let $f(x) = x\ln(x)$. Find the tangent line to the graph $y = f(x)$ at $x = 1$.

   **Numeric answer:** The tangent line is $y = x - 1$

4. In order to sell $q$ items, a company has to sell them at a price of $p = 100e^{-0.01q}$ dollars.

   a) What is the revenue function?

   **Numeric answer:** $R(q) = 100qe^{-0.01q}$

   b) What is the marginal revenue at $q = 10$ items?

   **Numeric answer:** $MR(10) = 81.44$ dollars/item

5. In order to sell $q$ items, a company has to sell them at a price of $p = 1000e^{-0.005q}$ dollars.

   a) What is the revenue function?

   **Numeric answer:** $R(q) = 1000qe^{-0.005q}$

   b) What is the marginal revenue at $q = 25$ items?

   **Numeric answer:** $MR(25) = 772.18$ dollars/item