

# Essentials of Calculus

## Homework 3.3

### The chain rule

1. Suppose  $f$  and  $g$  have values given by the following table

$x$	1	2	3	4	5
$f(x)$	5	3	4	9	1
$f'(x)$	5	-2	3	2	9
$g(x)$	4	5	1	2	4
$g'(x)$	-4	4	-2	3	2

and let  $h(x) = f(g(x))$ ,  $k(x) = g(f(x))$ . Evaluate the following expressions:

- a)  $h'(1)$
  - b)  $h'(2)$
  - c)  $h'(3)$
  - d)  $k'(1)$
  - e)  $k'(2)$
  - f)  $k'(3)$
2. Find  $f'(x)$  for the following  $f(x)$ :

- a)  $f(x) = (x^2 + 1)^5$
- b)  $f(x) = \sqrt{2x + 5}$
- c)  $f(x) = \frac{1}{(2x^2 - 2)^3}$
- d)  $f(x) = e^{x^2}$
- e)  $f(x) = e^{2x-4}$
- f)  $f(x) = e^{\sqrt{x}}$
- g)  $f(x) = \ln(x^2 - x)$
- h)  $f(x) = \ln(2x + 5)$
- i)  $f(x) = \frac{3}{\sqrt{x+2}} - 4e^{x^4}$

$$\text{j) } f(x) = 3(x^2 + 3x - 5)^3 + 2e^{x^2-4} - 5 \ln(2x^3 + 1)$$

$$\text{k) } f(x) = 5e^{2x-1} - 4 \ln(10x^3)$$

$$\text{l) } f(x) = 5x^3 + 7 - 3e^{x^3}$$

$$\text{m) } f(x) = \sqrt{x} + 2\sqrt{x^2 + 1} + 3\sqrt{e^x - 1}$$

3. Let  $f(x) = x^2 + 2x + 3e^{x-2}$ . Find an equation for the tangent line to  $y = f(x)$  at  $x = 2$ .
4. Let  $f(x) = \frac{2}{x^2+1} + 2$ . Find an equation for the tangent line to  $y = f(x)$  at  $x = 1$ .
5. In  $t$  seconds, an object will be  $f(t) = 5 + 2(t^2 + t)^4$  feet away. How fast will it be going in  $t = 2$  seconds?
6. The cost function for a company making  $q$  boxes of crayons is  $C(q) = 0.01q^2 + 20 \ln(2q + 1)$  dollars. What is the marginal cost at  $q = 10$  boxes?
7. The cost function for a company packaging  $q$  gallons of spring water is  $C(q) = 100 + 0.05\sqrt{q^4 + q}$  dollars. What is the marginal cost at  $q = 5$  gallons?