Essentials of Calculus

Homework 3.3

The chain rule

1. Suppose f and g have values given by the following table

x	1	2	3	4	5
$f(x)$	5	3	4	9	1
$f'(x)$	5	-2	3	2	9
$g(x)$	4	5	1	2	4
$g'(x)$	-4	4	-2	3	2

and let $h(x) = f(g(x))$, $k(x) = g(f(x))$. Evaluate the following expressions:

a) $h'(1)$

Numeric answer: $h'(1) = -8$

b) $h'(2)$

Numeric answer: $h'(2) = 36$

c) $h'(3)$

Numeric answer: $h'(3) = -10$

d) $k'(1)$

Numeric answer: $k'(1) = 10$

e) $k'(2)$

Numeric answer: $k'(2) = 4$

f) $k'(3)$

Numeric answer: $k'(3) = 9$

2. Find $f'(x)$ for the following $f(x)$:

a) $f(x) = (x^2 + 1)^5$

Numeric answer: $f'(x) = 10x(x^2 + 1)^4$

b) $f(x) = \sqrt{2x + 5}$

Numeric answer: $f'(x) = (2x + 5)^{-1/2}$

c) $f(x) = \frac{1}{(2x^2-2)^3}$

Numeric answer: $f'(x) = -12x(2x^2-2)^{-4}$

d) $f(x) = e^{x^2}$

Numeric answer: $f'(x) = 2xe^{x^2}$

e) $f(x) = e^{2x-4}$

Numeric answer: $f'(x) = 2e^{2x-4}$

f) $f(x) = e^{\sqrt{x}}$

Numeric answer: $f'(x) = \frac{1}{2}x^{-1/2}e^{\sqrt{x}}$

g) $f(x) = \ln(x^2 - x)$

Numeric answer: $f'(x) = \frac{2x-1}{x^2-x}$

h) $f(x) = \ln(2x+5)$

Numeric answer: $f'(x) = \frac{2}{2x+5}$

i) $f(x) = \frac{3}{\sqrt{x+2}} - 4e^{x^4}$

Numeric answer: $f'(x) = \frac{-3}{2}(x+2)^{-3/2} - 16x^3e^{x^4}$

j) $f(x) = 3(x^2+3x-5)^3 + 2e^{x^2-4} - 5\ln(2x^3+1)$

Numeric answer: $f'(x) = 9(2x+3)(x^2+3x-5)^2 + 4xe^{x^2-4} - \frac{30x}{2x^3+1}$

k) $f(x) = 5e^{2x-1} - 4\ln(10x^3)$

Numeric answer: $f'(x) = 10e^{2x-1} - \frac{120x^2}{10x^3}$

l) $f(x) = 5x^3 + 7 - 3e^{x^3}$

Numeric answer: $f'(x) = 15x^2 - 9x^2e^{x^3}$

m) $f(x) = \sqrt{x} + 2\sqrt{x^2+1} + 3\sqrt{e^x - 1}$

Numeric answer: $f'(x) = \frac{1}{2}x^{-1/2} + 2x(x^2+1)^{-1/2} + \frac{3}{2}e^x(e^x-1)^{-1/2}$

3. Let $f(x) = x^2 + 2x + 3e^{x-2}$. Find an equation for the tangent line to $y = f(x)$ at $x = 2$.

Numeric answer: The tangent line is $y - 11 = 9(x-4)$

4. Let $f(x) = \frac{2}{x^2+1} + 2$. Find an equation for the tangent line to $y = f(x)$ at $x = 1$.

   Numeric answer: The tangent line is $y - 11 = 9(x - 4)$

5. In t seconds, an object will be $f(t) = 5 + 2(t^2 + t)^4$ feet away. How fast will it be going in $t = 2$ seconds?

   Numeric answer: The velocity will be 8640 feet/sec.

6. The cost function for a company making q boxes of crayons is $C(q) = 0.01q^2 + 20\ln(2q + 1)$ dollars. What is the marginal cost at $q = 10$ boxes?

   Numeric answer: MC(10) = 2.10 dollars/box.

7. The cost function for a company packaging q gallons of spring water is $C(q) = 100 + 0.05\sqrt{q^4 + q}$ dollars. What is the marginal cost at $q = 5$ gallons?