# Essentials of Calculus <br> <br> Homework 3.2 <br> <br> Homework 3.2 <br> Exponentials and logarithms 

1. For each function $f(x)$, find $f^{\prime}(x)$.
a) $f(x)=10^{x}$

Numeric answer: $f^{\prime}(x)=\ln (10) 10^{x}$
b) $f(x)=2 x^{3}+3 \cdot 2^{x}$

Numeric answer: $f^{\prime}(x)=6 x^{2}+3 \ln (2) 2^{x}$
c) $f(x)=9 x^{4}-5 x+3-5 \cdot 2^{x}$

Numeric answer: $f^{\prime}(x)=36 x^{3}-5+5 \ln (2) 2^{x}$
d) $f(x)=5 \cdot 3^{x}+3 \cdot 5^{x}$

Numeric answer: $f^{\prime}(x)=5 \ln (3) 3^{x}+3 \ln (5) 5^{x}$
e) $f(x)=2 x^{3}-5 x+2 e^{x}$

Numeric answer: $f^{\prime}(x)=6 x^{2}-5+2 e^{x}$
f) $f(x)=6 e^{x}+6 x^{7}-3 \ln (x)$

Numeric answer: $f^{\prime}(x)=6 e^{x}+42 x^{6}-\frac{3}{x}$
g) $f(x)=5 \ln (x)-\frac{5}{x}$

Numeric answer: $f^{\prime}(x)=\frac{5}{x}+5 x^{-2}$
2. Let $f(x)=3 x^{2}+x-2 e^{x}$.
a) Find $f^{\prime}(0), f^{\prime}(1)$ and $f^{\prime}(2)$.

Numeric answer: $f^{\prime}(0)=-1$
$f^{\prime}(1)=7-2 e \approx 1.563$
b) Find an equation for the tangent line at $x=0$.

Numeric answer: $y=-2-x$
3. Let $f(x)=3 x^{2}+2 \ln (x)$. Find an equation for the tangent line at $x=1$.

Numeric answer: $y=3+8(x-1)$
4. If $\$ 100$ is put in a bank at $5 \%$ yearly interest, compounded continuous, in $t$ years there will be $f(t)=100 e^{0.05 t}$ dollars. Find $f(5)$ and $f^{\prime}(5)$, with units. Say what they represent.

Numeric answer: $f(5)=100 e^{0.25} \approx 128.4$ dollars $f^{\prime}(5)=5 e^{0.25} \approx 6.42$ dollars/year
5. A certain car is worth $f(t)=5000 e^{-0.05 t}$ dollars in $t$ years. Find $f(10)$ and $f^{\prime}(10)$, with units. Say what they represent. (Note: Apparently, a different problem was handed out. That problem and solution are at the end.)

Numeric answer: $f(10)=5000 e^{-0.5} \approx 3032$ dollars $f^{\prime}(10)=-250 e^{-0.5} \approx-151.63$ dollars/year
6. The human population of a certain region $t$ years from now is given by $P(t)=10000(.995)^{t}$. Find $P(50)$ and $P^{\prime}(50)$, with units, and give their interpretation.

Numeric answer: $P(50)=10000(0.995)^{50} \approx 7783$
people
$P^{\prime}(50)=10000\left(\ln (0.995)(0.995)^{50} \approx-39\right.$ people/year
7. It costs a company $C(q)=500+200 \ln (q)$ dollars to make $q$ objects. Find the total cost and marginal cost at a production level of $q=100$ objects.

Numeric answer: $C(100)=500+200 \ln (100) \approx$ 1421 dollars
$M C(100)=2$ dollars/object
A certain car is worth $f(t)=5000 e^{0.85 t}$ dollars in $t$ years. Find $f(10)$ and $f^{\prime}(10)$, with units. Say what they represent.

Numeric answer: $f(10)=5000 e^{8.5} \approx 24573844.20$ dollars. This is the amount the car will be worth in 10 years.
$f^{\prime}(10)=4250 e^{8.5} \approx 20887767.57$ dollars/year, which means that the car is gaining about 20887767.57 dollars a year in 50 years.

