Essentials of Calculus Homework 3.2 Exponentials and logarithms

1. For each function f(x), find f'(x).

a) $f(x) = 10^x$ Numeric answer: $f'(x) = \ln(10) 10^x$ b) $f(x) = 2x^3 + 3 \cdot 2^x$ **Numeric answer:** $f'(x) = 6x^2 + 3\ln(2)2^x$ c) $f(x) = 9x^4 - 5x + 3 - 5 \cdot 2^x$ Numeric answer: $f'(x) = 36x^3 - 5 + 5\ln(2)2^x$ d) $f(x) = 5 \cdot 3^x + 3 \cdot 5^x$ **Numeric answer:** $f'(x) = 5\ln(3)3^x + 3\ln(5)5^x$ e) $f(x) = 2x^3 - 5x + 2e^x$ Numeric answer: $f'(x) = 6x^2 - 5 + 2e^x$ f) $f(x) = 6e^x + 6x^7 - 3\ln(x)$ **Numeric answer:** $f'(x) = 6e^x + 42x^6 - \frac{3}{x}$ g) $f(x) = 5\ln(x) - \frac{5}{x}$ Numeric answer: $f'(x) = \frac{5}{x} + 5x^{-2}$ 2. Let $f(x) = 3x^2 + x - 2e^x$. a) Find f'(0), f'(1) and f'(2). Numeric answer: f'(0) = -1 $f'(1) = 7 - 2e \approx 1.563$ b) Find an equation for the tangent line at x = 0. **Numeric answer:** y = -2 - x

3. Let $f(x) = 3x^2 + 2\ln(x)$. Find an equation for the tangent line at x = 1.

Numeric answer: y = 3 + 8(x - 1)

4. If \$100 is put in a bank at 5% yearly interest, compounded continuous, in *t* years there will be $f(t) = 100e^{0.05t}$ dollars. Find f(5) and f'(5), with units. Say what they represent.

Numeric answer: $f(5) = 100e^{0.25} \approx 128.4$ dollars $f'(5) = 5e^{0.25} \approx 6.42$ dollars/year

5. A certain car is worth $f(t) = 5000e^{-0.05t}$ dollars in t years. Find f(10) and f'(10), with units. Say what they represent. (Note: Apparently, a different problem was handed out. That problem and solution are at the end.)

Numeric answer: $f(10) = 5000e^{-0.5} \approx 3032$ dollars $f'(10) = -250e^{-0.5} \approx -151.63$ dollars/year

6. The human population of a certain region t years from now is given by $P(t) = 10000(.995)^t$. Find P(50) and P'(50), with units, and give their interpretation.

Numeric answer: $P(50) = 10000(0.995)^{50} \approx 7783$ people $P'(50) = 10000(\ln(0.995)(0.995)^{50} \approx -39$ people/year

7. It costs a company $C(q) = 500 + 200 \ln(q)$ dollars to make q objects. Find the total cost and marginal cost at a production level of q = 100 objects.

Numeric answer: $C(100) = 500 + 200 \ln(100) \approx$ 1421 dollars MC(100) = 2 dollars/object

A certain car is worth $f(t) = 5000e^{0.85t}$ dollars in *t* years. Find f(10) and f'(10), with units. Say what they represent.

Numeric answer: $f(10) = 5000e^{8.5} \approx 24573844.20$ dollars. This is the amount the car will be worth in 10 years.

 $f'(10) = 4250e^{8.5} \approx 20887767.57$ dollars/year, which means that the car is gaining about 20887767.57 dollars a year in 50 years.