

# Essentials of Calculus

## Homework 2.3

### Interpretations of the derivative

1. Suppose that  $y = f(x)$  and  $f'(3) = 0.3$ .
  - a) If  $x$  increases by 0.5 ( $\Delta x = 0.5$ ), approximately how much does  $y$  change by (approximate  $\Delta y$ )?
  - b) If  $f(3) = 6$ , approximate the value of  $f(3.5)$ .
2. Suppose that  $\left.\frac{dy}{dt}\right|_{t=2} = -0.1$ .
  - a) If  $t$  decreases by 0.25 ( $\Delta t = -0.25$ ), approximately how much does  $y$  change by (approximate  $\Delta y$ )?
  - b) If  $y = 5$  when  $t = 2$ , approximate the value of  $y$  when  $t = 1.75$ .
3. Suppose that  $y = f(x)$  and  $f'(5) = -0.2$ .
  - a) If  $x$  increases by 0.5 ( $\Delta x = 0.5$ ), approximately how much does  $y$  change by (approximate  $\Delta y$ )?
  - b) If  $f(3) = 6$ , approximate the value of  $f(3.5)$ .
  - c) If  $f(10) = 15$  and  $f'(10) = 1.2$ , approximate  $f(11)$  and  $f(9.5)$ .
4. Suppose that  $f(t)$  represents the height (in feet) of a helium balloon in  $t$  seconds, and that  $f'(10) = 5$ .
  - a) What are the units for 10 and 5?
  - b) If  $f(10) = 200$ , about how high will be balloon be in 11 seconds?
5. Suppose that  $h(t)$  represents the height (in feet) of an airplane  $t$  hours after it takes off. What are the units of  $h'(t)$ ? What does it mean if  $h'(5) < 0$ ?

6. If a company sells  $x$  doodads, its revenue will be  $R$  dollars, and  $\frac{dR}{dx}\big|_{x=100} = 500$ .
- a) What are the units for 100 and 500?
  - b) If  $R(100) = 2500$ , about how much revenue will the company get if it sells 101 doodads? If it sells 99 doodads?
7. If business makes  $x$  thingamajigs, it will cost the company  $C$  dollars. We know that  $C(50) = 250$  and  $\frac{dC}{dx}\big|_{x=50} = 10$ .
- a) What are the units for 50, 250, and 10?
  - b) About how much will it cost the company to make 51 thingamajigs? To make 48 thingamajigs?
8. If a company makes  $x$  gizmos, it can sell them and make a profit of  $P$  dollars. If the company is currently making 200 gizmos and  $\frac{dP}{dx}\big|_{x=200} < 0$ , what should the company do?