Essentials of Calculus

Homework 2.3

Interpretations of the derivative

1. Suppose that $y = f(x)$ and $f'(3) = 0.3$.

   a) If x increases by 0.5 ($\Delta x = 0.5$), approximately how much does y change by (approximate Δy)?

   Numeric answer: $\Delta y \approx 0.15$

   b) If $f(3) = 6$, approximate the value of $f(3.5)$.

   Numeric answer: $f(3.5) \approx 6.15$

2. Suppose that $\left.\frac{dy}{dt}\right|_{t=2} = -0.1$.

   a) If t decreases by 0.25 ($\Delta t = -0.25$), approximately how much does y change by (approximate Δy)?

   Numeric answer: $\Delta y \approx 0.025$

   b) If $y = 5$ when $t = 2$, approximate the value of y when $t = 1.75$.

   Numeric answer: $y \approx 5.025$

3. Suppose that $y = f(x)$ and $f'(5) = -0.2$.

   a) If x increases by 0.5 ($\Delta x = 0.5$), approximately how much does y change by (approximate Δy)?

   Numeric answer: $\Delta y \approx -0.1$

   b) If $f(3) = 6$, approximate the value of $f(3.5)$.

   Numeric answer: $f(3.5) \approx 5.9$

   c) If $f(10) = 15$ and $f'(10) = 1.2$, approximate $f(11)$ and $f(9.5)$.

   Numeric answer: $f(11) \approx 16.2, f(9.5) \approx 14.4$

4. Suppose that $f(t)$ represents the height (in feet) of a helium balloon in t seconds, and that $f'(10) = 5$.

a) What are the units for 10 and 5?

b) If $f(10) = 200$, about how high will be balloon be in 11 seconds?

Numeric answer: The balloon will be about 205 feet high.

5. Suppose that $h(t)$ represents the height (in feet) of an airplane t hours after it takes off. What are the units of $h'(t)$? What does it mean if $h'(5) < 0$?

6. If a company sells x doodads, its revenue will be R dollars, and $\left.\frac{dR}{dx}\right|_{x=100} = 500$.

   a) What are the units for 100 and 500?

   b) If $R(100) = 2500$, about how much revenue will the company get if it sells 101 doodads? If it sells 99 doodads?

   Numeric answer: For selling 101 doodads, the revenue will be about 3000 dollars; for selling 99 doodads, the revenue will be about 2000 dollars.

7. If business makes x thingamajigs, it will cost the company C dollars. We know that $C(50) = 250$ and $\left.\frac{dC}{dx}\right|_{x=50} = 10$.

   a) What are the units for 50, 250, and 10?

   b) About how much will it cost the company to make 51 thingamajigs? To make 48 thingamajigs?

   Numeric answer: To make 51 thingamajigs, it will cost about 260 dollars; to make 48 thingamajigs, it will cost about 230 dollars.

8. If a company makes x gizmos, it can sell them and make a profit of P dollars. If the company is currently making 200 gizmos and $\left.\frac{dP}{dx}\right|_{x=200} < 0$, what should the company do?