## Essentials of Calculus

Homework 2.1 Rates of change

1. Let $f(x)=2 x^{2}-x$.
a) Approximate the average rate of change of $f$ from $x=3$ to $x=3.1$.

Numeric answer: $\frac{\Delta y}{\Delta x}=11.2$
b) Approximate the average rate of change of $f$ from $x=3$ to $x=3.05$.

Numeric answer: $\frac{\Delta y}{\Delta x}=11.1$
c) Approximate the average rate of change of $f$ from $x=3$ to $x=3.01$.

Numeric answer: $\frac{\Delta y}{\Delta x}=11.02$
d) Approximate $f^{\prime}(3)$.

Numeric answer: $f^{\prime}(3) \approx 11$
2. Let $f(x)=2^{x}$.
a) Approximate the average rate of change of $f$ from $x=1$ to $x=1.1$.

Numeric answer: $\frac{\Delta y}{\Delta x} \approx 1.435$
b) Approximate the average rate of change of $f$ from $x=1$ to $x=1.01$.

Numeric answer: $\frac{\Delta y}{\Delta x} \approx 1.391$
c) Approximate the average rate of change of $f$ from $x=1$ to $x=1.001$.

Numeric answer: $\frac{\Delta y}{\Delta x} \approx 1.387$
d) Approximate $f^{\prime}(1)$.

Numeric answer: $f^{\prime}(1) \approx 1.38$
3. A mouse is running directly away. In $t$ seconds, it is $f(t)=10-$ $2 t-0.5 t^{2}$ feet away.
a) Approximate the mouse's average velocity from $t=2$ to $t=$ 2.1 seconds.

Numeric answer: The average velocity will be about -4.05 feet/sec.
b) Approximate the average velocity from $t=2$ to $t=2.01$ seconds.

Numeric answer: The average velocity will be about -4.005 feet/sec.
c) Approximate the average velocity from $t=1.99$ to $t=2$ seconds.

Numeric answer: The average velocity will be about -3.995 feet/sec.
d) Approximate the mouse's velocity at $t=2$ seconds.

Numeric answer: The velocity will be about -4 feet/sec.
4. Suppose that $f$ has values given by the following table.

| $x$ | 0 | 0.5 | 1 | 1.5 |
| :--- | :--- | :--- | :--- | :--- |
| $f(x$ | 1 | 1.25 | 2 | 3.25 |

Approximate $f^{\prime}(1)$.
Numeric answer: $f^{\prime}(1) \approx 2.5$
5. Suppose that in $t$ hours, a biker has traveled the following distances (measured in miles).

| $t$ | 1 | 1.25 | 1.5 | 1.75 |
| :--- | :--- | :--- | :--- | :--- |
| distance | 20 | 25 | 31 | 35 |

Approximate the biker's velocity in 1.5 hours.
Numeric answer: The velocity will be about 16 miles/hour.
6. Suppose that the graph $y=f(x)$ looks like


For each value, state whether it is positive, negative or zero.
a) $f^{\prime}(-1)$

Numeric answer: $f^{\prime}(-1)<0$
b) $f^{\prime}(0)$

Numeric answer: $f^{\prime}(0)>0$
c) $f^{\prime}(1)$

Numeric answer: $f^{\prime}(1)=0$
d) $f^{\prime}(2)$

Numeric answer: $f^{\prime}(2)=0$
e) $f^{\prime}(3)$

Numeric answer: $f^{\prime}(3)<0$
7. Suppose that the graph $y=f(x)$ looks like


Approximate the following values:
a) $f^{\prime}(-1)$

Numeric answer: $f^{\prime}(-1) \approx 1$
b) $f^{\prime}(0)$

Numeric answer: $f^{\prime}(0) \approx 1 / 2$
c) $f^{\prime}(1)$

Numeric answer: $f^{\prime}(1) \approx-1$
d) $f^{\prime}(2)$

Numeric answer: $f^{\prime}(2) \approx 0$
e) $f^{\prime}(3)$

Numeric answer: $f^{\prime}(3) \approx-1$

